

Supergravity inspired Vector Curvaton

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It is investigated whether a massive Abelian vector field, whose gauge kinetic function is growing during inflation, can be responsible for the generation of the curvature perturbation in the Universe. Particle production is studied and it is shown that the vector field can obtain a scale invariant superhorizon spectrum of perturbations with a reasonable choice of kinetic function. After inflation the vector field begins coherent oscillations, during which it corresponds to pressureless isotropic matter. When the vector field dominates the Universe its perturbations give rise to the observed curvature perturbation following the curvaton scenario. It is found that this is possible if, after the end of inflation, the mass of the vector field increases at a phase transition at temperature of order 1 TeV or lower. Inhomogeneous reheating, whereby the vector field modulates the decay rate of the inflaton, is also studied.

I. INTRODUCTION

Observations suggest that the formation of structure in the Universe is due to the existence of a primordial spectrum of superhorizon curvature perturbations. The fact that they are superhorizon strongly supports the hypothesis that these perturbations were generated through an acausal process. The most compelling mechanism for this to date is cosmic inflation.

During inflation, all light, non-conformally invariant fields undergo particle production and obtain a superhorizon spectrum of perturbations. These perturbations can be responsible for the generation of the curvature perturbation in the Universe, if their spectrum is compatible with the observations. Traditionally, it has been considered that it is the perturbations of the inflaton field itself, which give rise to the curvature perturbation. However, this inflaton hypothesis typically results in overconstraining inflation model-building, which leads to fine-tuning.

For this reason, alternative suggestions have been recently put forward. According to such proposals, the field responsible for the curvature perturbation may have little or nothing to do with the dynamics of inflation. One possibility is to consider a field whose contribution to the density is negligible during inflation but, after the end of inflation, it manages to dominate (or nearly dominate) the Universe before its decay, thereby imposing its own curvature perturbation spectrum. This is the so-called curvaton hypothesis [1]. Under this hypothesis the fine-tuning problems of inflation are alleviated [2, 3], while one can attain inflation at low energy scales [4–6]. Many suggestions in the literature offer realistic candidates in theories beyond the standard model, which can play the role of the curvaton field.

Another suggestion along similar lines is that the field responsible for the curvature perturbation is not related to the dynamics of inflation but it affects the reheat-

ing process by modulating the decay rate of the inflaton. This is the inhomogeneous reheating mechanism [7, 8], which can also allow for low-scale inflation [9].

Until now the literature considers that the curvature perturbation in the Universe is due to particle production of a suitable *scalar field*, typically through one of the above mechanisms. However, even though theories beyond the standard model (in particular supersymmetric theories) contain a plethora of scalar fields, the fact that no scalar field has been observed as yet undermines somewhat the predictability and falsifiability of these models. In contrast, in this paper, we consider the possibility that the curvature perturbation is due to particle production of a *vector field* during inflation.

A massive vector field is non-conformally invariant and can indeed undergo particle production during inflation. In Ref. [10] this scenario has been investigated for a massive Abelian vector field. It was shown that a scale-invariant spectrum of perturbations can be generated provided the mass of the vector field satisfies the condition $m^2 \approx -2H_*^2$ during inflation, where H_* is the inflationary Hubble scale. However, this condition is hard to realise in a theoretically well motivated way.

This problem is overcome in this paper by considering a non-trivial evolution of the kinetic term for the vector field, during inflation. In supergravity the kinetic term of vector fields is determined, in general, by the gauge kinetic function which is a holomorphic function of the fields of the theory. We consider a similar setup here and assume that, the kinetic function is dominated by a degree of freedom which varies substantially during inflation, while the cosmological scales exit the horizon. We find that a scale-invariant spectrum of vector field perturbations can be attained, without the need for a negative mass-squared for the vector field, if the kinetic function is growing with time during inflation.

We then investigate how such a spectrum of vector field perturbations can give rise to the observed curvature perturbation in the Universe. In general, a homogeneous vector field generates an anisotropic pressure, which, if dominant, results in a large-scale anisotropy that contra-

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dicts the observations (isotropy of the CMB). This is why, the vector field cannot play the role of the inflaton (see, however, [11]). On the other hand, as shown in Ref. [10], a massive oscillating vector field has zero average pressure and behaves as pressureless, isotropic matter. Thus, it can safely dominate the Universe without generating a long-range anisotropy. Hence, one can employ the curvaton mechanism to generate the curvature perturbation in the Universe, using as curvaton a massive vector field, which has assumed a scale-invariant spectrum of perturbations during inflation. In this paper we study in detail the use of such a vector field as curvaton.

One other way to attempt to generate the curvature perturbation from the vector field without the latter ever dominating the Universe, is by considering that the vector field controls the decay rate of the inflaton, resulting in inhomogeneous reheating. We briefly investigate this scenario as well.

The paper is structured as follows. In Sec. II we derive the equations of motion for the perturbations of a massive vector field with a varying kinetic function and mass. In Sec. III we study particle production during inflation of this vector field and obtain the necessary conditions to attain the desired scale-invariant spectrum. In Sec. IV we study the dynamics of the scalar field which controls the kinetic function for our vector field. In Sec. V we obtain the spectrum of the produced perturbations in the case when the vector field has constant mass and also when its mass is controlled by the scalar field which also controls the kinetic function. In Sec. VI we study analytically the curvaton scenario. By obtaining the energy-momentum tensor for the vector field we find the scaling of its density during and after inflation and reheating. We then implement this to find the parameter space in which the vector field can generate the curvature perturbation. We find that the lower bound on the inflationary scale is too stringent to allow the scenario to work. In Sec. VII we employ the mass increment mechanism to lower further the inflationary scale. The mechanism assumes that the mass of the vector field grows at a phase transition after the end of inflation. In Sec. VIII we study possible complications to the scenario due to the dynamics and to particle production of the scalar field that controls the kinetic function. In Sec. IX we present a concrete example of our vector curvaton model, taking all the constraints into account. In Sec. X the inhomogeneous reheating mechanism is tried out, using as the inflaton the field that controls the kinetic function. The mechanism is shown to be ineffective. Finally, in Sec. XI we discuss our results and present our conclusions.

Throughout the paper we use natural units, where $c = \hbar = 1$ and Newton's gravitational constant is $8\pi G = m_P^{-2}$, with $m_P = 2.4 \times 10^{18} \text{ GeV}$ being the reduced Planck mass. The signature of the metric is (1,-1,-1,-1).

II. THE EQUATIONS OF MOTION

Consider the following Lagrangian density for a massive vector field with mass m

$$\mathcal{L} = -\frac{1}{4}f F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu, \quad (1)$$

where $f = f(t)$ is a function of cosmic time t reminiscent of the gauge kinetic function in supergravity.¹ In general, the mass of the vector field can also depend on time, i.e. $m = m(t)$. For an Abelian field, the field strength tensor is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (2)$$

Employing the above one obtains the field equations for the vector field:

$$\left[\partial_\mu + \left(\partial_\mu \ln \sqrt{-\mathcal{G}} \right) \right] [f(\partial^\mu A^\nu - \partial^\nu A^\mu)] + m^2 A^\nu = 0, \quad (3)$$

where \mathcal{G} is the determinant of the metric tensor.

Since we are interested in particle production during inflation we assume that, to a good approximation, the spacetime is spatially flat, homogeneous and isotropic. Hence, we use the flat-FRW metric:

$$ds^2 = dt^2 - a^2(t)dx^i dx^i, \quad (4)$$

where $a = a(t)$ is the scale factor of the Universe, x^i are Cartesian spatial coordinates with $i = 1, 2, 3$ and Einstein summation is assumed. Employing the above metric into Eq. (3) and following the process detailed in Ref. [10] we obtain the following temporal and spatial components of the field equations respectively:

$$\nabla \cdot \dot{\mathbf{A}} - \nabla^2 A_t + \frac{(am)^2}{f} A_t = 0, \quad (5)$$

and

$$\begin{aligned} \ddot{\mathbf{A}} + \left(H + \frac{\dot{f}}{f} \right) \dot{\mathbf{A}} + \frac{m^2}{f} \mathbf{A} - a^{-2} \nabla^2 \mathbf{A} = \\ = \left(\frac{\dot{f}}{f} - 2\frac{\dot{m}}{m} - 2H \right) \nabla A_t, \end{aligned} \quad (6)$$

where the dot denotes derivative with respect to the cosmic time and ∇ stands for the divergence or the gradient while $\nabla^2 \equiv \partial_i \partial_i$ is the Laplacian.

We expect inflation to homogenise the vector field and, therefore,

$$\partial_i A_\mu = 0 \quad \forall \quad \mu \in [0, 3]. \quad (7)$$

¹ A similar setup is employed in so-called dilaton electromagnetism [12], where $f = e^{-\lambda\Phi/m_P}$ with Φ being the dilaton. This setup has been used to break the conformality of electromagnetism and generate a primordial magnetic field during inflation [13] (see also [14]).

Enforcing this condition into Eq. (5) we obtain

$$A_t = 0. \quad (8)$$

Using Eqs. (7) and (8) into Eq. (6) we find

$$\ddot{\mathbf{A}} + \left(H + \frac{\dot{f}}{f}\right) \dot{\mathbf{A}} + \frac{m^2}{f} \mathbf{A} = 0. \quad (9)$$

The above is reminiscent of the Klein-Gordon equation of a homogeneous scalar field in an expanding Universe, with the crucial difference that the coefficient in the “friction” term does not feature a factor of $3H$.

We are interested in the generation of superhorizon perturbations of the vector field, which might be responsible for the curvature perturbations in the Universe. Therefore, we perturb the vector field around the homogeneous value $A_\mu(t)$ as follows:

$$\begin{aligned} A_\mu(t, \mathbf{x}) &= A_\mu(t) + \delta A_\mu(t, \mathbf{x}) \Rightarrow \\ \mathbf{A}(t, \mathbf{x}) &= \mathbf{A}(t) + \delta \mathbf{A}(t, \mathbf{x}) \quad \& \quad A_t(t, \mathbf{x}) = \delta A_t(t, \mathbf{x}), \end{aligned} \quad (10)$$

where we took into account Eq. (8). In the above $\mathbf{A}(t)$ satisfies Eq. (9). In view of Eqs. (9) and (10), Eqs. (5) and (6) become

$$\nabla \cdot (\delta \dot{\mathbf{A}}) - \nabla^2 \delta A_t + \frac{(am)^2}{f} \delta A_t = 0 \quad (11)$$

$$\begin{aligned} (\delta \ddot{\mathbf{A}}) + \left(H + \frac{\dot{f}}{f}\right) (\delta \dot{\mathbf{A}}) + \frac{m^2}{f} \delta \mathbf{A} - a^{-2} \nabla^2 \delta \mathbf{A} = \\ = \left(\frac{\dot{f}}{f} - 2\frac{\dot{m}}{m} - 2H\right) \nabla \delta A_t. \end{aligned} \quad (12)$$

Now, let us switch to momentum space by Fourier expanding the perturbations:

$$\delta A_\mu(t, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} \delta \mathcal{A}_\mu(t, \mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{x}). \quad (13)$$

Using the above, Eq. (11) becomes

$$\delta \mathcal{A}_t + \frac{i\partial_t(\mathbf{k} \cdot \delta \mathbf{A})}{k^2 + (am)^2/f} = 0, \quad (14)$$

where $k^2 \equiv \mathbf{k} \cdot \mathbf{k}$. Using this and Eq. (13) we can write Eq. (12) as

$$\begin{aligned} (\delta \ddot{\mathbf{A}}) + \left(H + \frac{\dot{f}}{f}\right) (\delta \dot{\mathbf{A}}) + \frac{m^2}{f} \delta \mathbf{A} + \left(\frac{k}{a}\right)^2 \delta \mathbf{A} + \\ + \left(2H + 2\frac{\dot{m}}{m} - \frac{\dot{f}}{f}\right) \frac{\mathbf{k} \partial_t(\mathbf{k} \cdot \delta \mathbf{A})}{k^2 + (am)^2/f} = 0. \end{aligned} \quad (15)$$

We can rewrite the above in terms of the components parallel and perpendicular to \mathbf{k} , defined as:

$$\delta \mathcal{A}^\parallel \equiv \frac{\mathbf{k}(\mathbf{k} \cdot \delta \mathbf{A})}{k^2} \quad \& \quad \delta \mathcal{A}^\perp \equiv \delta \mathbf{A} - \delta \mathcal{A}^\parallel. \quad (16)$$

Thus, we obtain the following equations of motion for the vector field perturbations in momentum space:

$$\left[\partial_t^2 + \left(H + \frac{\dot{f}}{f}\right) \partial_t + \frac{m^2}{f} + \left(\frac{k}{a}\right)^2 \right] \delta \mathcal{A}^\perp = 0 \quad (17)$$

$$\begin{aligned} \left\{ \partial_t^2 + \left[H + \frac{\dot{f}}{f} + \frac{\left(2H + 2\frac{\dot{m}}{m} - \frac{\dot{f}}{f}\right) k^2}{k^2 + \frac{(am)^2}{f}} \right] \partial_t + \right. \\ \left. + \frac{m^2}{f} + \left(\frac{k}{a}\right)^2 \right\} \delta \mathcal{A}^\parallel = 0. \end{aligned} \quad (18)$$

III. PARTICLE PRODUCTION

To investigate particle production during inflation for the vector field we need to solve the equation of motion for the perturbations of the field. The integration constants are then evaluated by matching the solution to the vacuum at early times (when $k/aH \rightarrow +\infty$), i.e. by demanding

$$\lim_{\frac{k}{aH} \rightarrow +\infty} \delta \mathcal{A}_k = \frac{1}{\sqrt{2k}} \exp(ik/aH), \quad (19)$$

where $\delta \mathcal{A}_k \equiv \delta \mathbf{A}(t, \mathbf{k})$ and we note that at early times the perturbation in question is well within the horizon, which means that $a \rightarrow 1$ and $k/aH \rightarrow kt$.

Afterwards we evaluate the solution at late times, when the perturbation is superhorizon in size (i.e. when $k/aH \rightarrow 0^+$). The power spectrum is obtained by

$$\mathcal{P}_{\mathcal{A}} = \frac{k^3}{2\pi^2} \left| \lim_{\frac{k}{aH} \rightarrow 0^+} \delta \mathcal{A}_k \right|^2. \quad (20)$$

We assume that, during inflation, H is constant. We also assume that f is proportional to some power of the scale factor, such that

$$f \propto a^{\alpha-1} \Rightarrow \frac{\dot{f}}{f} = (\alpha-1)H, \quad (21)$$

with α being a constant.

We will concern ourselves only with the transverse component of the vector field perturbations Eq. (17), whose equation of motion we write as

$$\left[\partial_t^2 + \alpha H \partial_t + \tilde{m}^2 + \left(\frac{k}{a}\right)^2 \right] \delta \mathcal{A}_k = 0, \quad (22)$$

where \tilde{m} is a constant associated with the mass m of the vector field (see below) and we have dropped the ‘ \perp ’ for simplicity.

Solving Eq. (22) and matching to the vacuum in Eq. (19), we obtain the solution

$$\delta \mathcal{A}_k = \frac{1}{2} \sqrt{\frac{\pi}{aH}} e^{i(\nu + \frac{1}{2})\frac{\pi}{2}} H_\nu^{(1)}(k/aH), \quad (23)$$

where with $H_\nu^{(1)}$ we denote the Hankel function of the first kind and

$$\nu \equiv \sqrt{\left(\frac{\alpha}{2}\right)^2 - \left(\frac{\tilde{m}}{H}\right)^2}. \quad (24)$$

The above solution at late times approaches

$$\lim_{\frac{k}{aH} \rightarrow 0^+} \delta \mathcal{A}_k = \frac{1}{2} \sqrt{\frac{\pi}{aH}} e^{i(\nu + \frac{1}{2})\frac{\pi}{2}} \times \\ \times \left[\frac{1 + i\pi(\frac{1}{2} - \nu)}{\Gamma(1 + \nu)} \left(\frac{k}{2aH}\right)^\nu - \frac{i}{\Gamma(1 - \nu)} \left(\frac{k}{2aH}\right)^{-\nu} \right]. \quad (25)$$

Hence, using Eq. (20) we find that the dominant contribution to the power spectrum is

$$\mathcal{P}_\mathcal{A} \approx \frac{4\pi}{|\Gamma(1 - \nu)|^2} \left(\frac{aH}{2\pi}\right)^2 \left(\frac{k}{2aH}\right)^{3-2\nu}. \quad (26)$$

Therefore, we may obtain a scale-invariant spectrum if

$$\nu \approx 3/2 \quad \Leftrightarrow \quad \left(\frac{\alpha}{2}\right)^2 \approx \frac{9}{4} + \left(\frac{\tilde{m}}{H}\right)^2. \quad (27)$$

In this case we find that a scale invariant spectrum of perturbations is recovered with

$$\mathcal{P}_\mathcal{A} \approx a^2 \left(\frac{H}{2\pi}\right)^2. \quad (28)$$

as in the case of a massless scalar field.

Parameterising the scale dependence of the perturbations in the usual manner

$$\mathcal{P}_\mathcal{A}(k) \propto k^{n_s-1}, \quad (29)$$

and comparing with Eq. (26) we obtain, for the spectral index, the result

$$n_s - 1 = 3 - 2\nu = 3 - \alpha \sqrt{1 - \left(\frac{2\tilde{m}}{\alpha H}\right)^2}, \quad (30)$$

where we also used Eq. (24). In the case when $\tilde{m} \ll H$ we find

$$n_s \simeq (4 - \alpha) - \frac{6}{\alpha}\eta \quad \text{where} \quad \eta \equiv \frac{1}{3} \left(\frac{\tilde{m}}{H}\right)^2, \quad (31)$$

which, when $\alpha = 3$, is the usual finding in the case of a light scalar field.²

² There is no contribution from $\epsilon \equiv -\dot{H}/H^2$ to the spectral index because we have taken $H = \text{const.}$

IV. FAST-ROLLING SCALAR FIELD

From Eq. (21) we see that, if $f = \text{constant}$ then $\alpha = 1$ and we can have a scale invariant spectrum of perturbations only if $\tilde{m}^2 \approx -2H^2$ (c.f Eq. (27)), i.e. only if the effective mass-squared of the vector field is negative [10]. To avoid this, we need to consider that $f(t)$ is controlled by a degree of freedom which undergoes non-trivial evolution during inflation, at least during the period when the cosmological scales exit the horizon. This is natural to expect in supergravity.

Indeed, in supergravity, the scalar fields of the theory receive a contribution to their mass of order the Hubble scale H during inflation, due to corrections to the scalar potential generated by a non-minimal Kähler potential [15]. Hence, these scalar fields are expected to evolve substantially during inflation as they fast-roll down the potential slopes. Hence, dependence of f on these scalar fields is expected to yield naturally $\dot{f} \neq 0$ during inflation. To parametrise this behaviour we assume that f is a function of some scalar field $\phi = \phi(t)$, whose value varies during inflation.

The gauge kinetic function in supergravity is a holomorphic function of the fields of the theory. Hence, we consider that $f(\phi)$ can be expanded around the origin as $f(\phi) \approx \sum_{1/2}^\infty c_n (\phi/M)^{2n}$, where M is some cutoff scale and c_n are constant coefficients. We assume that this sum is dominated by a term of n -th order, so that we can write

$$f(\phi) \simeq \left(\frac{\phi}{M}\right)^{2n}, \quad (32)$$

where we have absorbed c_n into M . Inserting the above into Eq. (21) we find

$$\phi \propto a^{\frac{\alpha-1}{2n}}. \quad (33)$$

Let us introduce the following Lagrangian density for the scalar field ϕ :

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi), \quad (34)$$

where, for the scalar potential, we consider

$$V(\phi) = V_0 - \frac{1}{2} m_\phi^2 \phi^2 + \dots, \quad (35)$$

where the dots denote higher order terms which stabilise the potential at $\phi_{\text{vev}} = M$, such that

$$V_0 \sim m_\phi^2 M^2. \quad (36)$$

Hence, the kinetic term of the vector field becomes canonical ($f = 1$) after ϕ settles at its vacuum expectation value (VEV).

Assuming that the field has been homogenised by inflation, its equation of motion, when $\phi < M$, is

$$\ddot{\phi} + 3H\dot{\phi} - m_\phi^2 \phi \simeq 0. \quad (37)$$

The solution of the above during inflation has a growing mode of the form

$$\phi \simeq \phi_0 \exp \left\{ \frac{3}{2} H \Delta t \left[\sqrt{1 + \frac{4}{9} \left(\frac{m_\phi}{H} \right)^2} - 1 \right] \right\}, \quad (38)$$

where ϕ_0 is the initial value at the onset of inflation and Δt is the elapsed time. Comparing the above with Eq. (33) we find

$$\frac{m_\phi}{H} = \frac{3}{2} \sqrt{\left(\frac{\alpha - 1}{3n} + 1 \right)^2 - 1}. \quad (39)$$

where we considered that $a \propto e^{H\Delta t}$. The above means that, if $\alpha, n = \mathcal{O}(1)$ then $m_\phi \sim H$ during inflation. This is naturally expected for scalar fields in supergravity due to corrections introduced to the scalar potential when considering a generic form of the Kähler potential [15]. This is the source of the so-called η -problem, which is endemic to inflation when a scalar field is used to produce the curvature perturbation in the Universe.

From Eq. (33) it is easy to obtain the number of e-folds it takes for ϕ to reach the minimum of $V(\phi)$:

$$N_\phi = \frac{2n}{\alpha - 1} \ln \left(\frac{M}{\phi_0} \right). \quad (40)$$

After inflation $H(t) < m_\phi$, which means that ϕ rushes toward its VEV, $\phi_{\text{vev}} = M$, in less than a Hubble time.

V. SPECTRUM OF PERTURBATIONS

In this section we concentrate on two particular possibilities, which may be realised in this model. Other possibilities exist but the following appear to be the most straightforward for investigation.

A. Constant mass

Suppose at first that the mass of the vector field is constant. In this case, the mass term in Eq. (17) is

$$\frac{m^2}{f} \propto a^{1-\alpha}. \quad (41)$$

Let us choose $\alpha = 3$.³ Then the above suggests that $m^2/f \propto a^{-2}$, which means that the mass term in Eq. (17) scales as the $(k/a)^2$ term. Thus, the resulting equation of

motion is of the form of Eq. (22) with $\alpha = 3$ and $\tilde{m} = 0$ under the substitution:

$$k \rightarrow k' \quad \text{where} \quad k' \equiv \sqrt{k^2 + k_c^2}, \quad (42)$$

where

$$k_c^2 \equiv \frac{(am)^2}{f} = (a_0 m)^2 \left(\frac{M}{\phi_0} \right)^{2n} = (a_0 m)^2 e^{2N_\phi}, \quad (43)$$

with a_0 being the value of the scale factor at the onset of inflation and we have used Eq. (40). The solution of Eq. (17) is, therefore, the one described in Eq. (23), which, at late times, approaches the result in Eq. (25), with $k \rightarrow k'$ and $\nu = 3/2$. Hence, in view of Eq. (20) we obtain the dominant contribution to the power spectrum:

$$\mathcal{P}_\mathcal{A} \approx \left(\frac{aH}{2\pi} \right)^2 \left(\frac{k^2}{k^2 + k_c^2} \right)^{3/2}. \quad (44)$$

Thus, when $k \gg k_c$, the power spectrum is approximately scale invariant. In the opposite case, $\mathcal{P}_\mathcal{A} \propto k^3$. If these perturbations are to give rise to the curvature perturbations in the Universe the cosmological scales should correspond to scales with $k_* > k_c$. Hence, we require:

$$\begin{aligned} \frac{k_c}{H} &= a_c < a_* \equiv a_{\text{end}} e^{-N_*} \\ \Rightarrow \frac{m}{H} &< \exp(N_{\text{tot}} - N_* - N_\phi), \end{aligned} \quad (45)$$

where a_c is the scale factor at the time when the scale k_c exits the horizon during inflation, a_{end} is the scale factor at the end of inflation, the subscript ‘*’ denotes the time when the cosmological scales exit the horizon, $N_{\text{tot}} \equiv a_{\text{end}}/a_0$ denotes the total number of e-folds of inflation and we have used Eq. (43).

The condition in Eq. (45) can be better understood when considering the ‘‘effective’’ mass of the vector field during inflation as featured in the equation of motion (9):

$$\begin{aligned} \frac{m^2}{f} &= m^2 e^{2N_\phi} \left(\frac{a_0}{a} \right)^2 \Rightarrow \\ \Rightarrow \frac{m}{\sqrt{f}} &= m \exp(N_\phi + N - N_{\text{tot}}), \end{aligned} \quad (46)$$

where we have used Eqs. (43) and (40) with $\alpha = 3$. In view of the above we see that the constraint in Eq. (45) corresponds to the requirement:

$$\left. \frac{m^2}{f} \right|_* < H. \quad (47)$$

If ϕ were also responsible for inflation we would have $N_{\text{tot}} = N_\phi$ and the above constraint would read $m < e^{-N_*} H$. According to Ref. [10], satisfying this bound allows the generation of a scale-invariant perturbation spectrum for the longitudinal component of the vector field, which may also be used to generate the curvature perturbation in the Universe. However, as discussed in Ref. [10], such a bound on the mass of the vector field is very hard to satisfy. Hence, most probably, ϕ

³ Another interesting choice is $\alpha = 1$ because, in this case $m^2/f = \text{const.}$ However, by virtue of Eq. (33), such a choice implies that $\phi = \text{const.}$ which means that $f = \text{const.}$ (c.f. Eq. (32)). This case, therefore, is already explored in Ref. [10] and requires a negative mass-squared for the vector field.

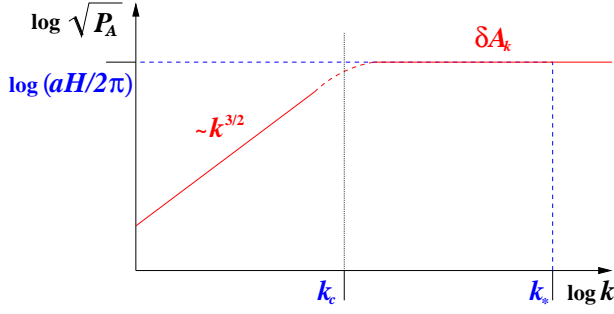


FIG. 1: Illustration of the superhorizon spectrum of the transverse component of the perturbation of a sufficiently light vector field in the case when $m = \text{const.}$ and $f \propto a^2$. At momenta smaller than k_c the spectrum is $\delta A_k \propto k^{3/2}$, while when $k \gg k_c$ the spectrum becomes approximately scale invariant $\delta A_k \approx aH/2\pi$. Hence, the cosmological scales should correspond to momentum $k_* > k_c$.

needs to be some scalar field other than the inflaton (see also footnote 6). In this case too, though, we need inflation not to last too long because the cosmological scales have to exit the horizon while ϕ is still rolling. Otherwise, the roll of ϕ is irrelevant and we are back to the case studied in Ref. [10].

B. Higgsed vector field

Suppose, now that the mass of the vector field is due to an interaction between the former and the scalar field ϕ . In this case, the Lagrangian of the model is

$$\mathcal{L} = -\frac{1}{4}f(\phi)F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi(D^\mu\phi)^* - V(\phi), \quad (48)$$

where $V(\phi)$ is given by Eq. (35), $D_\mu\phi = \partial_\mu\phi - igA_\mu\phi$ is the covariant derivative in field space and g is the (gauge) coupling (ϕ is taken to be real for simplicity). Then, the mass term of the vector field is

$$\delta\mathcal{L} = \frac{1}{2}g^2\phi^2 A_\mu A^\mu, \quad (49)$$

i.e. the mass of the vector field, in this case, is $m = g\phi$. Consequently, this time, the mass term in Eq. (17) is

$$\frac{m^2}{f} \propto \phi^{2(1-n)} \propto a^{(\frac{n-1}{n})(1-\alpha)}, \quad (50)$$

where we used Eqs. (32) and (33).

Now, one might be interested to obtain a scale invariant spectrum in the same manner as the previous subsection, i.e. by taking $m^2/f \propto a^{-2}$. As we have seen, this case corresponds to $\tilde{m} = 0$ in Eq. (22). Then, according to Eq. (27), scale invariance requires $\alpha = 3$. However, in view of Eq. (50), this is only possible when $n \gg 1$, which is not realistic. Thus, it seems that $m^2/f \propto a^{-2}$ is not realisable in this case.

Another option is to consider that $m^2/f = \text{const.}$ From Eq. (50) we see that this is possible if either $\alpha = 1$ or $n = 1$. The former case implies that $\phi = \text{const.}$ (cf. Eq. (33)), which means that $f = \text{const.}$ (cf. Eq. (32)). This case has been explored in Ref. [10] and requires a negative mass-squared for the vector field. Let us then concentrate in the latter case, when $n = 1$.

Assuming $n = 1$ means that Eq. (9) becomes

$$\ddot{\mathbf{A}} + \alpha H \dot{\mathbf{A}} + (gM)^2 \mathbf{A} = 0, \quad (51)$$

where we have used Eq. (21) and that $m = g\phi$ with

$$f = \frac{\phi^2}{M^2}. \quad (52)$$

Therefore, the mass term in Eq. (17) becomes

$$\frac{m^2}{f} = (gM)^2, \quad (53)$$

which suggests that $\tilde{m} = gM$ in this case. Hence, according to Eq. (27), scale invariance requires

$$\left(\frac{\alpha}{2}\right)^2 \approx \frac{9}{4} + \left(\frac{gM}{H}\right)^2, \quad (54)$$

which means that $\alpha \geq 3$.

In view of the above, the solution to Eq. (51), during inflation, is

$$|\mathbf{A}| = C_1 e^{-\frac{1}{2}(\alpha+3)H\Delta t} + C_2 e^{\frac{1}{2}(3-\alpha)H\Delta t} \propto a^{\frac{3-\alpha}{2}}, \quad (55)$$

where C_1 and C_2 are constants of integration and, in the proportionality relation, we have considered only the “growing mode”. Hence we see that, for $\alpha > 3$, the magnitude of the vector field is decreasing. This is undesirable, as will be made clear later (see footnote 5). Hence, we choose $\alpha = 3$, in which case $|\mathbf{A}| \simeq \text{const.}$ To satisfy, therefore, Eq. (54), we need to enforce the constraint

$$gM < \frac{3}{2}H. \quad (56)$$

The above constraint is necessary in order to obtain an approximately scale invariant spectrum of perturbations. However, if these perturbations are to account for the curvature perturbations in the Universe then the above constraint is tightened further by spectral index considerations. Indeed, from Eq. (30) we readily find

$$n_s - 1 \simeq \frac{2}{3} \left(\frac{gM}{H}\right)^2. \quad (57)$$

Hence, the spectrum obtained is blue in contrast to the observational preferences. Since $n_s \approx 1.00$ is still marginally acceptable and the precision of the observational data is at the level of a few percent, we obtain the following bound

$$gM \lesssim 0.1 H. \quad (58)$$

VI. VECTOR CURVATON

One mechanism for generating the curvature perturbation in the Universe starting from a superhorizon spectrum of vector field perturbations, follows the curvaton scenario. In this case, the vector field, while subdominant during inflation, may come to dominate (or nearly dominate) some time afterwards. When it does so, it imposes its own curvature perturbation onto the Universe [1].

A. The energy momentum tensor

To compute if and when the vector field dominates the Universe, in order to imprint its superhorizon perturbation spectrum, we follow the evolution of the energy-momentum tensor of the vector field.

Using Eq. (1), the energy momentum tensor for A_μ is

$$T_{\mu\nu} = f \left(\frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} - F_{\mu\rho} F_{\nu}^{\rho} \right) + m^2 \left(A_\mu A_\nu - \frac{1}{2} g_{\mu\nu} A_\rho A^\rho \right). \quad (59)$$

Assume that the homogenised vector field lies along the z -direction

$$A_\mu = (0, 0, 0, A(t)). \quad (60)$$

Then the energy-momentum tensor can be written in the form

$$T_\mu^\nu = \text{diag}(\rho_A, -p_\perp, -p_\perp, +p_\perp), \quad (61)$$

where

$$\rho_A \equiv \rho_{\text{kin}} + V_A, \quad p_\perp \equiv \rho_{\text{kin}} - V_A, \quad (62)$$

with

$$\rho_{\text{kin}} \equiv -\frac{1}{4} f F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} a^{-2} f \dot{A}^2, \quad (63)$$

$$V_A \equiv -\frac{1}{2} m^2 A_\mu A^\mu = \frac{1}{2} a^{-2} m^2 A^2. \quad (64)$$

From Eq. (61) we see that the energy momentum tensor for our vector field resembles the one of a perfect fluid, with the crucial difference that the pressure along the longitudinal direction is of opposite sign to the pressure along the transverse directions. Thus, if the pressure is non-zero and the vector field dominates the Universe, then large scale anisotropy will be generated. This is the reason we did not consider that A_μ can play the role of the inflaton field in the first place.

However, in Ref. [10] it was shown that, once $m > H$, the vector field undergoes quasi-harmonic oscillations, during which $\overline{\rho_{\text{kin}}} \approx \overline{V_A}$, where the overline denotes average over a large number of oscillations.⁴ This result

suggests that $\overline{p_\perp} \approx 0$ and the oscillating vector field behaves as isotropic pressureless matter. Therefore, it can indeed dominate the radiation background, *without introducing a large scale anisotropy*.

In Ref. [10] it was indeed confirmed that, during the oscillations, the density of the vector field scales as

$$\overline{\rho_A} \propto a^{-3}. \quad (65)$$

How does the density of the vector field scale before the onset of the oscillations? By virtue of Eqs. (47), (53) and (58) we have that,

$$\frac{m^2}{f} < H \quad (66)$$

during inflation. Then, it can be easily shown that Eq. (9) suggests that $A \equiv |A|$ remains frozen. Hence, $\rho_{\text{kin}} \propto \dot{A}^2 \rightarrow 0$, while

$$\rho_A \simeq V_A \propto a^{-2}, \quad (67)$$

where we considered Eq. (64). As shown in Ref. [10], the scaling of the vector density remains as such after inflation (when $f = 1$) as well, provided $m < H(t)$. Thus, we see that, despite the fact that A is frozen before the onset of the oscillations, the density of the vector field decreases.

B. Curvaton Physics

Using the results in the previous section we can trace the evolution of the density of the vector field during and after inflation. As noted above, to avoid a large scale anisotropy, we need that the vector field begins oscillating before its decay and before it dominates the Universe. Thus, we require:

$$\Gamma, m > \Gamma_A, H_{\text{dom}}, \quad (68)$$

where Γ and Γ_A are the decay rates of the inflaton field and the vector curvaton field respectively and the subscript ‘dom’ denotes the time when the curvaton dominates the Universe (if it does not decay earlier). Let us define the density parameter of the vector field as

$$\Omega \equiv \frac{\rho_A}{\rho}, \quad (69)$$

where ρ is the background density typically corresponding to either the oscillating inflaton field or the thermal bath of its decay products.

In the standard picture, after the end of inflation the inflaton field undergoes quasi-harmonic oscillations until it decays at reheating. During these coherent oscillations the inflaton corresponds to a collection of massive particles (inflavons) which behave like pressureless matter. Hence, for the background density in this period we have $\rho \propto a^{-3}$. After the decay of the inflaton (when $\Gamma \geq H(t)$)

⁴ Note that, since after inflation $\phi = M$ and $f = 1$, the treatment and the results of Ref. [10] are directly applicable here.

the Universe becomes dominated by the relativistic decay products, in which case $\rho \propto a^{-4}$. In view of the above and Eqs. (65) and (67) it is easy to obtain the density parameter of the vector field at the onset of its oscillations (denoted by ‘osc’):

$$\Omega_{\text{osc}} \sim \Omega_{\text{end}} \left(\frac{H_{\text{end}}}{m} \right)^{2/3} \min \left\{ 1, \frac{m}{\Gamma} \right\}^{-1/3}. \quad (70)$$

Similarly, if the curvaton decays before domination, we obtain

$$\Omega_{\text{dec}} \sim \Omega_{\text{end}} \left(\frac{H_{\text{end}}}{m} \right)^{2/3} \left(\frac{\Gamma}{\Gamma_A} \right)^{1/2} \min \left\{ 1, \frac{m}{\Gamma} \right\}^{1/6}, \quad (71)$$

where ‘dec’ denotes the time of the vector field decay ($H_{\text{dec}} = \Gamma_A$). Finally, if the curvaton dominates the Universe before its decay (i.e. $H_{\text{dom}} > \Gamma_A$) we find

$$H_{\text{dom}} \sim \Omega_{\text{end}}^2 \Gamma \left(\frac{H_{\text{end}}}{m} \right)^{4/3} \min \left\{ 1, \frac{m}{\Gamma} \right\}^{1/3}. \quad (72)$$

Let us now estimate Ω_{end} . Since during inflation $A \simeq \text{const.}$ we have $\rho_A \simeq V_A = \frac{1}{2}m^2(A/a)^2$. Hence, using that $\rho_{\text{end}} = 3H_{\text{end}}^2 m_P^2$, we obtain

$$\Omega_{\text{end}} \sim e^{-2N_{\text{tot}}} \left(\frac{m}{H_{\text{end}}} \right)^2 \left(\frac{W_0}{m_P} \right)^2, \quad (73)$$

where $W_0 \equiv A/a_0$ is the magnitude of the *physical* vector field at the onset of inflation.

In Ref. [10] it was explained that A_μ is the *comoving* vector field, which has the expansion of the Universe factored out. In a homogeneous and isotropic Universe the associated *physical* vector field is

$$\mathbf{W} \equiv \mathbf{A}/a. \quad (74)$$

This can be understood easily by considering the mass term in the Lagrangian in Eq. (1). Using the flat FRW metric in Eq. (4) one has

$$\delta\mathcal{L} = \frac{1}{2}m^2 A_\mu A^\mu = \frac{1}{2}m^2 (A_t^2 - a^{-2} A_i A_i). \quad (75)$$

Since the Lagrangian corresponds to a physical (observable) quantity we readily see that the spatial components of the physical vector field are A_i/a , as in Eq. (74). Note also, that this is the explanation of the explicit appearance of the scale factor in the results shown in Eqs. (28), (63) and (64). For example, in view of Eq. (20), the value of the scale invariant power spectrum of the physical vector field W_μ is $\mathcal{P}_W = \mathcal{P}_A/a^2 = (H/2\pi)^2$, i.e. identical to the case of a massless scalar field [10].

From the above we see that, even though A is frozen during inflation, $W \equiv |\mathbf{W}| = A/a$ is gradually decreasing, which explains the exponential suppression of Ω_{end} in Eq. (73).

C. The curvature perturbation

The curvature perturbation associated with the vector field is

$$\zeta_A = -H \frac{\delta\rho_A}{\dot{\rho}_A} = \frac{1}{3} \frac{\delta\rho_A}{\rho_A} \Big|_{\text{dec}}, \quad (76)$$

where we considered that, before its decay, the vector field is undergoing coherent oscillations, for which $\dot{\rho}_A = -3H\rho_A$ as suggested by Eq. (65). Since during oscillations we have $\overline{\rho_A} \approx 2\overline{V_A} = a^{-2}m^2\overline{A^2}$, we find

$$\zeta_A = \frac{\delta\rho_A}{3\rho_A} \Big|_{\text{dec}} \simeq \frac{2}{3} \frac{\delta\hat{A}}{\hat{A}} \Big|_{\text{dec}} \approx \frac{2}{3} \frac{\delta A}{A} \Big|_{\text{osc}}, \quad (77)$$

where we took into account that, during the oscillations, both δA and A obey the same equation of motion, since Eq. (9) is linear. We also considered that $\overline{A^2} \approx \frac{1}{2}\hat{A}^2$, where by \hat{A} we denote the amplitude of the oscillations, which is equal to A at the onset of the oscillations.

Before the onset of the oscillations we have $m/f < H$, which means that A is frozen. However, as evident from Eq. (28), δA grows as $\delta A \propto a$. That is, *although the spectrum of the perturbations of the vector field is scale invariant, its amplitude grows with the scale factor of the Universe.* This implies that

$$\frac{\delta A}{A} \Big|_{\text{osc}} = \frac{a_{\text{osc}}}{a_*} \frac{\delta A}{A} \Big|_* = \left(\frac{a_{\text{osc}}}{a_*} \right) \frac{a_* H_*}{2\pi A_*} \approx \frac{H_*}{2\pi W_{\text{osc}}}, \quad (78)$$

where we have used that $W_{\text{osc}} \equiv (A/a)_{\text{osc}} \approx A_*/a_{\text{osc}}$ and we have assumed that $\delta A/A < 1$ at all times. The above shows that the growth of the amplitude of the perturbations before the onset of the oscillations is due to the decrease of the physical vector field, according to Eq. (74). Using Eq. (74), it is easy to find

$$W_{\text{osc}} \sim W_0 e^{-N_{\text{tot}}} \left(\frac{m}{H_{\text{end}}} \right)^{2/3} \min \left\{ 1, \frac{m}{\Gamma} \right\}^{-1/6}, \quad (79)$$

where we assumed that A is frozen throughout inflation. Putting together Eqs. (77), (78) and (79) we obtain

$$\zeta_A \sim e^{N_{\text{tot}}} \frac{H_*}{W_0} \left(\frac{H_{\text{end}}}{m} \right)^{2/3} \min \left\{ 1, \frac{m}{\Gamma} \right\}^{1/6}. \quad (80)$$

D. The parameter space

Substituting from the above $e^{N_{\text{tot}}}$ into Eq. (73) we get

$$\Omega_{\text{end}} \sim \zeta_A^{-2} \left(\frac{H_*}{m_P} \right)^2 \left(\frac{m}{H_{\text{end}}} \right)^{2/3} \min \left\{ 1, \frac{m}{\Gamma} \right\}^{1/3}, \quad (81)$$

which, remarkably, is independent of W_0 . Plugging Eq. (81) into Eqs. (71) and (72) we find that, if the vector curvaton decays before domination

$$\Omega_{\text{dec}} \sim \zeta_A^{-2} \left(\frac{H_*}{m_P} \right)^2 \left(\frac{\Gamma}{\Gamma_A} \right)^{1/2} \min \left\{ 1, \frac{m}{\Gamma} \right\}^{1/2}, \quad (82)$$

while if the vector curvaton dominates the Universe before its decay

$$H_{\text{dom}} \sim \Gamma \zeta_A^{-4} \left(\frac{H_*}{m_P} \right)^4 \min \left\{ 1, \frac{m}{\Gamma} \right\}. \quad (83)$$

Solving Eqs. (82) and (83) for H_* we obtain

$$\frac{H_*}{m_P} \sim \frac{\zeta}{\sqrt{\Omega_{\text{dec}}}} \left(\frac{\max\{H_{\text{dom}}, \Gamma_A\}}{\min\{\Gamma, m\}} \right)^{1/4}, \quad (84)$$

where we used the fact that, in the curvaton mechanism $\zeta \sim \Omega_{\text{dec}} \zeta_A$, where $\zeta \simeq 5 \times 10^{-5}$ is the observed curvature perturbation. Now, considering that $\Omega_{\text{dec}} \lesssim 1$, $\max\{H_{\text{dom}}, \Gamma_A\} \geq \Gamma_A$ and $m \leq 0.1 H_*$, it can be easily verified that

$$\left(\frac{H_*}{m_P} \right)^5 \geq 10 \zeta^4 \frac{\Gamma_A}{m_P}. \quad (85)$$

The lower bound in the above is attained when $\Gamma \geq m$, $m \rightarrow 0.1 H_*$, $\Omega_{\text{end}} \rightarrow 1$ and $H_{\text{dom}} \rightarrow \Gamma_A$. This case corresponds to almost prompt reheating and curvaton decay as soon as the latter dominates the Universe.

Demanding that the decay of the curvaton occurs before Big Bang Nucleosynthesis (BBN) imposes the bound $\Gamma_A > T_{\text{BBN}}^2/m_P$, which suggests

$$\frac{H_*}{m_P} > 10^{1/5} \zeta^{4/5} \left(\frac{T_{\text{BBN}}}{m_P} \right)^{2/5} \Rightarrow H_* > 10^6 \text{ GeV}, \quad (86)$$

where $T_{\text{BBN}} \simeq 1 \text{ MeV}$ is the temperature at BBN. Hence, under this mechanism, the inflationary energy scale cannot be lower than $V_*^{1/4} \sim 10^{12} \text{ GeV}$, which agrees with the generic bound for the curvaton mechanism [16].

However, there is an important subtlety that needs to be considered here. Even though W_0 drops out from the calculations, one still must take into account the evolution of $W = A/a$ during inflation. This is because, in the above, we have assumed that W decreases as $W \propto a^{-1}$ since A is frozen. However, because $\mathcal{P}_{\mathcal{W}} \simeq (H_*/2\pi)^2$, the decrease of W will be halted if $W \lesssim H_*$. Thus, we need to postulate that $W_{\text{end}} > H_{\text{end}} \approx H_*$.⁵ For this we need to obtain an estimate of N_{tot} .

As discussed above, the parameter space for H_* is maximised if the vector curvaton decays when it is about to dominate the density of the Universe. This means that, after the decay of the inflaton field, the Universe remains radiation dominated, in which case, the number of e-folds corresponding to the horizon at present is

$$N_H \simeq 67 - \frac{1}{2} \ln \left(\frac{m_P}{H_*} \right) + \ln \left(\frac{H_*}{\Gamma} \right). \quad (87)$$

The parameter space for H_* is maximised when $\Gamma \sim H_*$, which also results in minimising N_H . Now, postulating $W_{\text{end}} = e^{-N_{\text{tot}}} W_0 > H_*$ and considering $N_{\text{tot}} > N_H$ (so that inflation solves the horizon and flatness problems) we obtain the bound: $W_0 > 10^8 m_P$! Such huge values of W_0 are unacceptably unrealistic. If, on the other hand, we demand $W_0 \lesssim m_P$ then we find that the bound $W_{\text{end}} > H_{\text{end}}$ requires

$$N_{\text{tot}} \leq \ln \left(\frac{W_0}{H_*} \right) \lesssim \ln \left(\frac{m_P}{H_*} \right). \quad (88)$$

In view of Eqs. (86) and (87), the above bound cannot satisfy $N_{\text{tot}} > N_H$, i.e. inflation is not enough to solve the horizon and flatness problems. It seems, therefore, that some modification is required, which will allow low scale inflation, for the vector curvaton scenario to work.

VII. MASS INCREMENT

In Ref. [4] the possibility of low scale inflation in the context of the curvaton mechanism was investigated. It was shown that this is indeed possible in two ways. One possibility is to consider as curvaton a pseudo-Nambu Goldstone boson, whose order parameter increases after the cosmological scales exit the horizon during inflation. This mechanism was implemented in Ref. [6] and it was shown that inflation with H_* at least as low as 1 TeV was possible to attain. The other technique involves a phase transition after the end of inflation, which gives rise to a sudden increment of the curvaton's mass (see also Ref. [5]). It is this mechanism that we attempt to implement in this paper to the case of a vector curvaton.

We assume, therefore, that a phase transition takes place at some time after the end of inflation but before the onset of the vector field oscillations. The mass of the vector field is increased from m to m_0 at this phase transition to become larger than the Hubble scale at the time, so that oscillations begin immediately. Hence, the phase transition corresponds to Hubble scale $m < H_{\text{osc}} \leq m_0$.

A. Relaxing the bound on the inflationary scale

The sudden increment of the mass of the vector field results in a corresponding growth of the density of the vector field. Since, $\rho_A \simeq V_A \propto m^2$ before the oscillations (c.f. Eq. (64)), we find that Ω_{osc} grows by a factor of $(m_0/m)^2$. Hence, we have

$$\Omega_{\text{osc}} \sim \Omega_{\text{end}} \left(\frac{m_0}{m} \right)^2 \left(\frac{H_{\text{end}}}{H_{\text{osc}}} \right)^{2/3} \min \left\{ 1, \frac{H_{\text{osc}}}{\Gamma} \right\}^{-1/3}. \quad (89)$$

The above is directly obtainable by Eq. (70) with the substitution $m \rightarrow H_{\text{osc}}$ and taking also the growth factor $(m_0/m)^2$ into account. An important constraint here is that the increment of the density of the vector field

⁵ Note that this bound can be much more stringent if A is not frozen but diminishes with time. This is why we have chosen $\alpha = 3$ in Sec. V B.

does not surpass the overall density available at the phase transition, i.e.

$$\Omega_{\text{osc}} \leq 1. \quad (90)$$

Using the above, in the case when the curvaton decays before domination, we obtain

$$\begin{aligned} \Omega_{\text{dec}} &\sim \Omega_{\text{end}} \left(\frac{m_0}{m}\right)^2 \left(\frac{H_{\text{end}}}{H_{\text{osc}}}\right)^{2/3} \left(\frac{\Gamma}{\Gamma_A}\right)^{1/2} \times \\ &\times \min\left\{1, \frac{H_{\text{osc}}}{\Gamma}\right\}^{1/6}, \end{aligned} \quad (91)$$

while in the case when the curvaton dominates the Universe before its decay

$$H_{\text{dom}} \sim \Omega_{\text{end}}^2 \Gamma \left(\frac{m_0}{m}\right)^4 \left(\frac{H_{\text{end}}}{H_{\text{osc}}}\right)^{4/3} \min\left\{1, \frac{H_{\text{osc}}}{\Gamma}\right\}^{1/3}. \quad (92)$$

Note that, in the above, Ω_{end} is still given by Eq. (73).

It is easy to see that Eqs. (79) and (80) remain unaffected by the mass increment, apart from the substitution $m \rightarrow H_{\text{osc}}$. Thus, we have

$$W_{\text{osc}} \sim W_0 e^{-N_{\text{tot}}} \left(\frac{H_{\text{osc}}}{H_{\text{end}}}\right)^{2/3} \min\left\{1, \frac{H_{\text{osc}}}{\Gamma}\right\}^{-1/6}, \quad (93)$$

and

$$\zeta_A \sim e^{N_{\text{tot}}} \frac{H_*}{W_0} \left(\frac{H_{\text{end}}}{H_{\text{osc}}}\right)^{2/3} \min\left\{1, \frac{H_{\text{osc}}}{\Gamma}\right\}^{1/6}. \quad (94)$$

Then, working as in the previous section, we obtain

$$\begin{aligned} \Omega_{\text{end}} &\sim \zeta_A^{-2} \left(\frac{H_*}{m_P}\right)^2 \left(\frac{H_{\text{osc}}}{H_{\text{end}}}\right)^{2/3} \left(\frac{m}{H_{\text{osc}}}\right)^2 \times \\ &\times \min\left\{1, \frac{H_{\text{osc}}}{\Gamma}\right\}^{1/3}, \end{aligned} \quad (95)$$

which is the equivalent of Eq. (81). Using this we find that, if the vector curvaton decays before domination

$$\begin{aligned} \Omega_{\text{dec}} &\sim \zeta_A^{-2} \left(\frac{m_0}{H_{\text{osc}}}\right)^2 \left(\frac{H_*}{m_P}\right)^2 \left(\frac{\Gamma}{\Gamma_A}\right)^{1/2} \times \\ &\times \min\left\{1, \frac{H_{\text{osc}}}{\Gamma}\right\}^{1/2}, \end{aligned} \quad (96)$$

while if the vector curvaton dominates the Universe before its decay

$$H_{\text{dom}} \sim \Gamma \zeta_A^{-4} \left(\frac{m_0}{H_{\text{osc}}}\right)^4 \left(\frac{H_*}{m_P}\right)^4 \min\left\{1, \frac{H_{\text{osc}}}{\Gamma}\right\}. \quad (97)$$

Solving Eqs. (96) and (97) for H_* we obtain

$$\frac{H_*}{m_P} \sim \frac{\zeta}{\sqrt{\Omega_{\text{dec}}}} \left(\frac{\max\{H_{\text{dom}}, \Gamma_A\}}{\min\{\Gamma, H_{\text{osc}}\}}\right)^{1/4} \frac{H_{\text{osc}}}{m_0}, \quad (98)$$

where we used the fact that, in the curvaton mechanism $\zeta \sim \Omega_{\text{dec}} \zeta_A$. Comparing the above with Eq. (84) we see that, apart from the substitution $m \rightarrow H_{\text{osc}}$, there is an extra factor of H_{osc}/m_0 in the right-hand-side. This means that, if $m_0 \gg H_{\text{osc}}$, the lower bound on H_* can be substantially relaxed to the desired level.

In contrast to the previous section, due to the extra factor of H_{osc}/m_0 , the lower bound on H_* can be more relaxed the *latter* the oscillations begin. Hence, the lowest bound is found when $H_{\text{osc}} \rightarrow \Gamma_A$. Indeed, in this case it is easy to find

$$\frac{H_*}{m_P} > \zeta \frac{\Gamma_A}{m_0}. \quad (99)$$

The above shown bound corresponds to $\Omega_{\text{end}} \rightarrow 1$ and $\Gamma \geq H_{\text{osc}} > \Gamma_A \geq H_{\text{dom}}$, i.e. to the case when the phase transition (which results in the oscillations of the vector curvaton) takes place just before the latter decays and as soon as it dominates the Universe. Since we need some oscillations before the curvaton domination and decay, in order to avoid a long-range anisotropy, the above lower bound is unattainable.

B. Additional bound on the inflationary scale

The decay rate of the vector curvaton is

$$\Gamma_A \sim h^2 m_0 \quad \text{with} \quad \frac{m_0}{m_P} \lesssim h \lesssim 1 \quad (100)$$

where the lower bound to the decay coupling h corresponds to gravitational decay. Using this we have

$$\frac{\max\{H_{\text{dom}}, \Gamma_A\}}{H_{\text{osc}}} \geq \frac{\Gamma_A}{H_{\text{osc}}} \gtrsim \left(\frac{m_0}{m_P}\right)^2 \frac{m_0}{H_{\text{osc}}}. \quad (101)$$

Inserting the above into Eq. (98) and after a little algebra we obtain

$$\frac{H_*}{m_P} \gtrsim \frac{\zeta^2}{\Omega_{\text{dec}}} \frac{H_{\text{osc}}}{H_*} \sqrt{\frac{H_{\text{osc}}}{m_0}} \max\left\{1, \frac{H_{\text{osc}}}{\Gamma}\right\}^{1/2}, \quad (102)$$

where the lower bound is attained when the vector curvaton decays gravitationally.

Now, from Eqs. (90) and (95) we get

$$\frac{m_0}{m_P} \lesssim \frac{\zeta}{\Omega_{\text{dec}}} \frac{H_{\text{osc}}}{H_*}, \quad (103)$$

where we used $\zeta_A \sim \Omega_{\text{dec}} \zeta$. The upper bound corresponds to the case when the density of the oscillating vector field dominates the Universe immediately after the phase transition. Using the above, Eq. (102) results in the bound

$$\frac{H_*}{m_P} \gtrsim \frac{\zeta^3}{\Omega_{\text{dec}}} \left(\frac{H_{\text{osc}}}{H_*}\right)^2 \max\left\{1, \frac{H_{\text{osc}}}{\Gamma}\right\}. \quad (104)$$

The above bound suggests that the mass increment mechanism can relax the lower bound on H_* *only if the*

phase transition occurs much later than the end of inflation. To show this, consider the opposite case, when $H_{\text{osc}} \sim H_* \geq \Gamma$. In this case, and considering also that $\Omega_{\text{dec}} \leq 1$, we find

$$H_* \gtrsim \zeta^3 m_P \sim 10^5 \text{ GeV}, \quad (105)$$

which is not too different from the bound in Eq. (86).

C. The parameter space revisited

Let us investigate now whether, under the mass increment mechanism, it is possible to achieve enough e-folds of inflation to solve the horizon and flatness problems while generating the observed amplitude for the curvature perturbation. To maximise the parameter space, we assume $\Omega_{\text{dec}} \rightarrow 1$, i.e. $\zeta_A \rightarrow \zeta$. Also, we consider $\Gamma_A \geq H_{\text{dom}}$, which means that the vector curvaton decays as soon as it dominates the Universe. Finally, since the bounds on the inflationary scale are relaxed for small values of H_{osc} , we assume $\Gamma \geq H_{\text{osc}}$, that is the phase transition occurs after the decay of the inflaton field.

Under the above assumptions Eq. (94) gives

$$e^{N_{\text{tot}}} \sim \zeta \frac{W_0}{H_*} \left(\frac{H_{\text{osc}}}{H_*} \right)^{2/3} \left(\frac{\Gamma}{H_{\text{osc}}} \right)^{1/6}, \quad (106)$$

while the bound in Eq. (104) can be written as

$$\frac{H_*}{m_P} \gtrsim \zeta \left(\frac{H_{\text{osc}}}{m_P} \right)^{2/3}. \quad (107)$$

Now, writing Eq. (87) as

$$e^{N_H} \sim 10^{29} \left(\frac{H_*}{m_P} \right)^{1/2} \frac{H_*}{\Gamma} \quad (108)$$

and using Eqs. (106) and (107), the requirement $N_{\text{tot}} \geq N_H$ results in the bound:

$$H_{\text{osc}} \lesssim 10^{-30} \left(\frac{W_0}{m_P} \right)^{6/5} \left(\frac{\Gamma}{H_*} \right)^{7/5} m_P. \quad (109)$$

Taking $W_0 \sim m_P$ and also $\Gamma \sim H_*$ (prompt reheating) we find that the phase transition, which results to the growth of the mass of the vector field, can take place at temperature

$$T_{\text{osc}} \lesssim 1 \text{ TeV}. \quad (110)$$

The above upper bound can be saturated when the bound in Eq. (107) is saturated, i.e. when the vector curvaton decays gravitationally.

Thus, we see that it is indeed possible to attain enough inflation to solve the horizon and flatness problems and explain the curvature perturbations in the Universe, when the phase transition, which results in mass increment for the vector field, occurs around the time of the breakdown of electroweak unification.

VIII. SCALAR FIELD CONCERNS

Apart from the above considerations there are a couple of issues regarding the scalar field ϕ , whose evolution is crucial during inflation, since it controls $f(\phi)$.

A. Production of ϕ during inflation

One issue that needs to be examined is whether ϕ also manages to obtain a superhorizon spectrum of perturbations and, if so, whether they may give rise to an acceptable or not contribution to the curvature perturbation.

Being tachyonic, ϕ is guaranteed to undergo particle production during inflation. One can understand this as follows. From Eq. (37) one obtains the following equation of motion for the Fourier modes of the perturbation $\delta\phi$ of the field

$$\left[\partial_t^2 + 3H\partial_t - m_\phi^2 + \left(\frac{k}{a} \right)^2 \right] \delta\varphi_k = 0, \quad (111)$$

where

$$\delta\phi(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \delta\varphi_k(t, \mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}}. \quad (112)$$

Solving Eq. (111) with vacuum boundary conditions in the same manner as in Sec. III one obtains the following power spectrum

$$\mathcal{P}_\phi \approx \frac{4\pi}{|\Gamma(1-\nu)|^2} \left(\frac{H}{2\pi} \right)^2 \left(\frac{k}{2aH} \right)^{3-2\nu} \quad (113)$$

where

$$\nu \equiv \sqrt{\frac{9}{4} + \left(\frac{m_\phi}{H} \right)^2} = \frac{3}{2} \left(\frac{1}{n} + 1 \right), \quad (114)$$

where we used Eq. (39), taking $\alpha = 3$ as discussed in Sec. V. From Eq. (113) it is evident that a scale invariant spectrum is attainable only if $\nu \approx 3/2$. However, as suggested by Eq. (114), such a spectrum is attainable only if n is very large. For example, if $n = 1$, as discussed in Sec. VB, then $\nu = 5/2$ and $\mathcal{P}_\phi \propto k^{-2}$. If such a spectrum of perturbations contributed significantly to the curvature perturbation then it would be incompatible with the observations.

The contribution of the perturbations of ϕ to the curvature perturbation is

$$\delta\zeta \sim \Omega_\phi \zeta_\phi \lesssim \Omega_\phi, \quad (115)$$

where we considered that $\zeta_\phi \lesssim 1$ and also defined the density parameter of ϕ as

$$\Omega_\phi \equiv \frac{\rho_\phi}{\rho} \sim \left(\frac{M}{m_P} \right)^2, \quad (116)$$

where we used that, during inflation $\rho_\phi \sim V_0 \sim (m_\phi M)^2$ [c.f. Eq. (36)] and $(m_\phi/H)^2 = \frac{9}{4n} \left(\frac{1}{n} + 2 \right) \sim \mathcal{O}(1)$, according to Eq. (39).

We need to make sure that ϕ does not produce an excessive curvature perturbation compared to the observations, which suggest $\zeta \simeq 5 \times 10^{-5}$. Thus, avoiding conflict with observations is guaranteed if $\delta\zeta \lesssim \zeta$, i.e.

$$M \lesssim \sqrt{\zeta} m_P \sim 2 \times 10^{16} \text{ GeV}. \quad (117)$$

Note that, as the roll of ϕ towards M progresses, the associated curvature perturbation $\zeta_\phi \propto \delta\phi/\phi$ is diminished, not only because ϕ grows but also because the spectrum of $\delta\phi$ is red. Hence, the above bound on M can be relaxed if the cosmological scales exit the horizon after the initial outburst of tachyonic perturbations has subsided somewhat.⁶

B. Source terms in the field equation of ϕ

The dependence on ϕ of the vector field kinetic term gives rise to source terms in the field equation of the scalar field. To study their influence let us consider the following Lagrangian density:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}f(\phi)F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2A_\mu A^\mu + \\ & + \frac{1}{2}D_\mu\phi(D^\mu\phi)^* - V(\phi), \end{aligned} \quad (118)$$

where $V(\phi)$ is given by Eq. (35) and $D_\mu\phi = \partial_\mu\phi - igA_\mu\phi$. The case of constant mass corresponds to $g = 0$, while the Higgsed vector field case corresponds to $m = 0$. From the above we find

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} - m_\phi^2\phi &= -\frac{1}{4}f'(\phi)F_{\mu\nu}F^{\mu\nu} + g^2A_\mu A^\mu \Rightarrow \\ \ddot{\phi} + 3H\dot{\phi} &= \\ \left\{ m_\phi^2 - a^{-2} \left[(gA)^2 - n \left(\frac{\phi}{M} \right)^{2(n-1)} \left(\frac{\dot{A}}{M} \right)^2 \right] \right\} \phi, \end{aligned} \quad (119)$$

where the prime denotes derivative with respect to ϕ and, in the last line of the above we have used Eq. (32) as well as that $A_\mu A^\mu = -a^{-2}A^2$ and also $F_{\mu\nu}F^{\mu\nu} = -2a^{-2}\dot{A}^2$.

Assuming $\alpha = 3$ as discussed in Sec. V, Eq. (39) suggests $m_\phi = \frac{\sqrt{1+3n}}{n}H \sim H$. On the other hand, due to Eqs. (47), (53) and (58), we have $m^2/f < H$, which means that $A \equiv |A|$ is frozen, since the mass term in Eq. (9) is negligible compared to the “friction” term $3H\dot{A}$. This implies that $\dot{A} \rightarrow 0$, which means that Eq. (119) can be recast as

$$\ddot{\phi} + 3H\dot{\phi} + (g^2W^2 - m_\phi^2)\phi \simeq 0, \quad (120)$$

where we used also Eq. (74). Now, in the constant mass case $g = 0$ and, therefore, the above equation reduces to Eq. (37). In the Higgsed vector field case, though, this is not necessarily so. Indeed, due to Eq. (58), we have $gW \lesssim 0.1H(W/M)$, which might still dominate $m_\phi \sim H$, if $W > \mathcal{O}(10)M$. Since, in principle, $W \leq W_0 \lesssim m_P$ this is not impossible, given Eq. (117).

What happens if $gW > m_\phi$? Note, at first, that, since A is frozen during inflation, $W \propto a^{-1}$, i.e. W is decreasing exponentially, which means that eventually m_ϕ becomes dominant, whatever the initial value of W . Still, just after the onset of inflation we may well have $gW_0 \gg m_\phi \sim H$. According to Eq. (120), a positive mass-squared larger than H would rapidly send ϕ to the origin. Consequently, since $m = 0$ in Eq. (118) in the Higgsed vector field case, the vector field is rendered exactly massless. This means that conformal invariance is restored and no perturbations of the vector field are generated [10].

As W decreases, however, the effective mass-squared of ϕ : $m_{\text{eff}}^2 \equiv g^2W^2 - m_\phi^2$ becomes smaller than H^2 , in which case particle production of ϕ generates a condensate for ϕ of order $\langle\phi^2\rangle \sim (H/2\pi)^2$. Indeed, Eq. (114) becomes $\nu = \sqrt{\frac{9}{4} + (m_\phi/H)^2 - (gW/H)^2}$. Hence, particle production begins when $(gW/H)^2 < \frac{9}{4} + \frac{1+3n}{n^2}$, where we used Eq. (39) with $\alpha = 3$. After $m_{\text{eff}}^2 < 0$, a phase transition sends ϕ rolling off the origin and down the potential hill in Eq. (35) as described in Sec. IV, while Eq. (120) reduces to Eq. (37).

Thus, when $gW_0 > H_*$, there is an initial period of inflation, where there is no vector particle production, while ϕ is sent to the origin. This period lasts for

$$N_W = \ln \left(\frac{gW_0}{H_*} \right) \quad (121)$$

e-foldings. Afterwards, a phase transition occurs which releases ϕ from the origin, the conformal invariance of the vector field is broken and particle production takes place as discussed in Sec. VB. From the above we see that the Higgs vector field case has the advantage of explaining the initial condition of ϕ on top of the potential hill, if W_0 is large enough.

IX. A CONCRETE EXAMPLE

To visualise the above findings we briefly study a particular example, considering the case of a Higgsed vector field. Thus, the Lagrangian density is given in Eq. (48). We take $\alpha = 3$ and $n = 1$, that is we assume that $f(\phi)$ is given by Eq. (52). According to Eq. (33) $\phi \propto a$, while Eq. (40) suggests

$$N_\phi = \ln \left(\frac{M}{\phi_0} \right). \quad (122)$$

In order to obtain an approximately scale invariant spectrum of perturbations we have to take the constraint in

⁶ Note that, if $M \ll m_P$ then ϕ cannot play the role of the inflaton because $V_0 \ll V_*$, where we used Eq. (36) and also that $m_\phi \sim H_*$, according to Eq. (39).

Eq. (58) into account. We assume that this constraint well satisfied, so that

$$gM \ll 0.1 H. \quad (123)$$

For the scalar field, which controls the mass of the vector field $m(\phi) = g\phi$, we consider a Higgs-type potential

$$V(\phi) = \frac{1}{4}\lambda(\phi^2 - M^2)^2, \quad (124)$$

with λ being a constant. In view of the above potential and also of Eq. (39), we have

$$m_\phi = \sqrt{\lambda}M = 2H_*. \quad (125)$$

From Eqs. (123) and (125) we readily obtain

$$g \ll \sqrt{\lambda}/20. \quad (126)$$

We assume that reheating is prompt and also that the vector curvaton decays as soon as it dominates the Universe. This means

$$\Gamma \sim H_* \quad \text{and} \quad \Omega_{\text{dec}} \sim 1 \quad \text{and} \quad \Gamma_A \geq H_{\text{dom}}. \quad (127)$$

Also, we assume that the vector curvaton decays through gravitational interactions, i.e.

$$\Gamma_A \sim \frac{m_0^3}{m_P^2}. \quad (128)$$

Furthermore, we assume that $W_0 \sim m_P$. This means that the bound in Eq. (109) becomes $H_{\text{osc}} \lesssim 10^{-30}m_P$.

We choose the following value for our example

$$H_{\text{osc}} \sim 10^{-32}m_P \Rightarrow T_{\text{osc}} \sim 100 \text{ GeV}, \quad (129)$$

i.e. the phase transition which results in the increment of the mass of the vector field occurs at the breaking of electroweak unification.

Similarly, we choose the decay rate of the vector curvaton to be

$$\Gamma_A \sim 10^{-36}m_P \Rightarrow T_{\text{dec}} \sim 1 \text{ GeV} \gg T_{\text{BBN}}. \quad (130)$$

From Eqs. (128) and (130) we find

$$m_0 \sim 10^{-12}m_P \sim 10^6 \text{ GeV}. \quad (131)$$

Now, in view of Eq. (127), Eqs. (89) and (95) give

$$\Omega_{\text{osc}} \sim \zeta^{-2} \left(\frac{H_*}{m_P} \right)^2 \left(\frac{m_0}{H_{\text{osc}}} \right)^2. \quad (132)$$

Similarly, Eq. (98) becomes

$$\frac{H_*}{m_P} \sim \zeta \left(\frac{\Gamma_A}{H_{\text{osc}}} \right)^{1/4} \frac{H_{\text{osc}}}{m_0}. \quad (133)$$

Combining the above we find

$$\Omega_{\text{osc}} \sim \left(\frac{\Gamma_A}{H_{\text{osc}}} \right)^{1/2} \sim \frac{T_{\text{dec}}}{T_{\text{osc}}} \sim 10^{-2}, \quad (134)$$

which satisfies the bound in Eq. (90). The above estimate for Ω_{osc} is quite reasonable, assuming equipartition of energy at the phase transition over a large number $[\mathcal{O}(10^2)]$ of degrees of freedom. This argument substantiates our choice of Γ_A in Eq. (130).

Using Eqs. (129), (130) and (131), Eq. (133) gives

$$H_* \sim 10^{-26}m_P \Rightarrow V_*^{1/4} \sim 10^5 \text{ GeV}. \quad (135)$$

Hence, we have low scale inflation. This means that reheating, even though prompt, will not result in gravitino overproduction. Also, one typically expects that the contribution of the inflaton to the curvature perturbation is negligible.

Inserting Eq. (135) into Eq. (104) and considering also Eqs. (127) and (129) it can be easily shown that the bound in Eq. (104) is saturated. This is expected since we assumed that the vector curvaton decays gravitationally. Similarly, it can be checked that the bound in Eq. (103) is also satisfied.

Employing Eq. (135) into Eq. (108) we find

$$N_H \simeq 37, \quad (136)$$

where we used also Eq. (127). Similarly, using Eqs. (127), (129) and (135), Eq. (106) gives

$$N_{\text{tot}} \simeq 41, \quad (137)$$

where we have used $W_0 \sim m_P$. Thus we see that $N_{\text{tot}} > N_H$ as required for the solution of the horizon and flatness problems. It is easy also to confirm that the bound in Eq. (88) is well satisfied.

In order to attain a scale-invariant spectrum of perturbations over cosmological scales up to the horizon at present we need to satisfy the constraint:

$$N_W < N_{\text{tot}} - N_H, \quad (138)$$

which ensures that the backreaction of the vector field onto ϕ becomes negligible before the current horizon scale exits the horizon during inflation. In view of Eqs. (121), (136) and (137), the above results in the bound

$$g < 10^{-24}, \quad (139)$$

where we considered $W_0 \sim m_P$. Hence, we see that the interaction between the vector field and ϕ must be quite suppressed.

Furthermore, the curvature perturbation spectrum must extend down to scales at least as small as the horizon at the time of matter-radiation equality $t_{\text{eq}} \sim 10^4 \text{ yrs}$. Thus, the e-fold range must be at least

$$\Delta N_{\text{obs}} = \frac{2}{3} \ln \left(\frac{t_0}{t_{\text{eq}}} \right) \simeq 9, \quad (140)$$

where $t_0 \sim 10 \text{ Gyrs}$ is the age of the Universe.⁷ As discussed in Sec. III, the generation of vector field perturbations ceases when ϕ assumes its VEV: $\phi \rightarrow M$ and

⁷ The recent dark energy domination of the Universe corresponds to less than an e-fold and can be ignored.

$f(\phi) \rightarrow 1$. After this moment $\alpha \rightarrow 1$ and $\nu \approx 1/2$, which, according to Eq. (26), gives $\mathcal{P}_A = (k/2\pi)^2$. This is the vacuum spectrum as can be readily confirmed by Eqs. (19) and (20). Hence, after N_ϕ e-folds of inflation, particle production stops. Therefore, in order to ascertain that the produced spectrum of perturbations extends over the entire range of the cosmological scales, we need to impose

$$N_\phi > (N_{\text{tot}} - N_H) + \Delta N_{\text{obs}} \simeq 13. \quad (141)$$

Since there is a period of N_W e-folds after the onset of inflation, during which the backreaction of the vector field sends ϕ to the origin, we can safely assume that, after the end of this period, the ϕ field begins to roll down its potential (c.f. Eq. (124)) with initial value $\phi_0 \simeq H_*/2\pi$, as determined by its quantum fluctuations. Using this, Eqs. (122) and (141) result in the constraint

$$10^{-3} \text{ GeV} < M \lesssim 10^{16} \text{ GeV}, \quad (142)$$

where the upper bound is due to Eq. (117).

From Eqs. (117) and (139) we also find

$$m = gM < 10^{-8} \text{ GeV} \sim H_*, \quad (143)$$

where we also considered Eq. (135). Hence, the vector field is indeed light during inflation. The upper bound on m is much more stringent though, due to the requirement $m < H_{\text{osc}} \ll H_*$. Indeed, using Eqs. (129) and (135) we find

$$\frac{m}{H_*} < 10^{-6}. \quad (144)$$

In view of Eq. (57) and considering that $m = gM$, we find that $n_s \approx 1$ to a high accuracy, provided the contribution from $\epsilon \equiv -\dot{H}/H^2$ is negligible. This value is marginally acceptable in terms of the observations. Since the data prefer a lower value, however, one may assume a large-field inflation model, with non-negligible ϵ . In this case, in accordance to the curvaton scenario [1], we have

$$n_s - 1 \simeq -2\epsilon \quad (145)$$

For example, with quadratic chaotic inflation, one finds

$$2\epsilon(N_H) = \frac{2}{1 + 2N_H} \simeq 0.03, \quad (146)$$

which gives the spectral index $n_s \simeq 0.97$ and the tensor fraction $r = 12.4\epsilon \simeq 0.33$, which is more acceptable by the latest WMAP data [17].

Let us choose, for illustrative purposes, $M \sim 1 \text{ TeV}$, which lies comfortably within the allowed range in Eq. (142). In this case, Eqs. (125) and (135) suggest $\sqrt{\lambda} \sim 10^{-11}$, which is in agreement with Eqs. (126) and (139). Also, Eq. (122) gives $N_\phi \simeq 25 > \Delta N_{\text{obs}}$, as required.

X. INHOMOGENEOUS REHEATING

In this section we briefly discuss an altogether different possibility from the curvaton mechanism for the use of a vector field to generate the curvature perturbation in the Universe. This is the inhomogeneous reheating mechanism, first introduced in Ref. [7]. According to this mechanism the curvature perturbations are due to the modulation of the decay rate of the inflaton field, because of its interaction with another field, which carries a superhorizon spectrum of perturbations. In our setup, one might employ this idea using the ϕ field in the Higgsed vector case as an inflaton, whose decay rate is modulated by the perturbations of the vector field.

According to the modulated reheating mechanism, the resulting curvature perturbation is related with the modulation of the decay rate of the inflaton as follows [7, 8]:

$$\zeta \sim \kappa \frac{\delta\Gamma}{\Gamma} \Big|_{\text{reh}}, \quad (147)$$

where $\kappa \sim 0.1$. We must, therefore, estimate the modulation of Γ at reheating.

The decay rate of the inflaton field ϕ is of the order

$$\Gamma \sim \hat{h}^2 m_{\text{inf}}, \quad (148)$$

where \hat{h} is the coupling of the inflaton field to its decay products. Now, for the inflaton mass we have

$$m_{\text{inf}}^2 = m_\phi^2 - g^2 A_\mu A^\mu = m_\phi^2 + g^2 W^2, \quad (149)$$

where we used that $A_\mu A^\mu = -a^{-2} A^2 \equiv -W^2$ according to Eqs. (60) and (74). From Eqs. (147), (148) and (149) we find

$$\zeta \sim \kappa \frac{\delta m_{\text{inf}}}{m_{\text{inf}}} \Big|_{\text{reh}} \sim \kappa \left[1 + \left(\frac{m_\phi}{gW_{\text{reh}}} \right)^2 \right]^{-1} \frac{\delta W}{W} \Big|_{\text{reh}}, \quad (150)$$

where ‘reh’ denotes the time of reheating.

As discussed in Sec. VIA, before the oscillations $\rho_A \propto a^{-2}$, while during the oscillations $\bar{\rho}_A \propto a^{-3}$. Since, in both cases $\rho_A \sim V_A \propto W^2$ (c.f. Eq. (64)) we find

$$W \propto \begin{cases} a^{-1} & \text{for } H > m \\ a^{-3/2} & \text{for } H \leq m \end{cases}. \quad (151)$$

Using this, it is easy to obtain

$$\frac{\delta W}{W} \Big|_{\text{reh}} = \frac{\delta A}{A} \Big|_{\text{reh}} = \min \left\{ 1, \frac{\Gamma}{m} \right\} \frac{H_*}{2\pi W_{\text{reh}}}, \quad (152)$$

where we used that, after the onset of the oscillations $\delta A/A$ remains constant and also that $\delta W = \sqrt{\mathcal{P}_W} = a^{-1} \sqrt{\mathcal{P}_A} = H_*/2\pi$ as suggested by Eq. (28).

Combining Eqs. (150) and (152) one gets

$$\zeta \sim \kappa \min \left\{ 1, \frac{\Gamma}{m} \right\} \left[1 + \left(\frac{m_\phi}{gW_{\text{reh}}} \right)^2 \right]^{-1} \frac{H_*}{2\pi W_{\text{reh}}}. \quad (153)$$

Case 1: Suppose, at first, that

$$gW_{\text{reh}} < m_\phi . \quad (154)$$

Then, Eq. (153) becomes

$$gW_{\text{reh}} \sim \frac{2\pi}{g} \left(\frac{\zeta}{\kappa} \right) \frac{m_\phi^2}{H_*} \min \left\{ 1, \frac{\Gamma}{m} \right\}^{-1} \quad (155)$$

Case 2: Suppose, now, that

$$gW_{\text{reh}} \geq m_\phi . \quad (156)$$

Then, Eq. (153) becomes

$$gW_{\text{reh}} \sim \frac{g}{2\pi} \left(\frac{\zeta}{\kappa} \right)^{-1} H_* \min \left\{ 1, \frac{\Gamma}{m} \right\} \quad (157)$$

Combining Eqs. (154) and (156) with Eqs. (155) and (157) respectively, we find that, in all cases

$$g \geq 2\pi \left(\frac{\zeta}{\kappa} \right) \frac{m_\phi}{H_*} \min \left\{ 1, \frac{\Gamma}{m} \right\}^{-1} . \quad (158)$$

Since in the Higgsed vector case (Sec. VB) a scale invariant perturbation spectrum requires $\alpha = 3$ and $n = 1$, Eq. (39) suggests that $m_\phi = 2H_*$. Using this and also that $\kappa \sim 0.1$, the above results in

$$g \gtrsim 10^{-3} \quad (159)$$

However, combining this with Eq. (58) suggests

$$H_* \gtrsim 10^{-2} m_P , \quad (160)$$

where $M \sim m_P$ is the VEV of the inflaton field ϕ , as implied by the fact that $V_* = V_0$ (c.f. Eq. (36)). Since $\alpha = 3$ and $n = 1$, using Eq. (40) we obtain

$$N_{\text{tot}} = N_\phi = \ln \left(\frac{M}{\phi_0} \right) \leq \ln \left(\frac{2\pi m_P}{H_*} \right) \lesssim 6 , \quad (161)$$

where we have considered that the initial value for the inflaton cannot be smaller than its quantum fluctuation $\phi_0 \geq H_*/2\pi$. The above number of e-folds is far too small to compare with the requirements for the solution of the horizon problem. Indeed, from Eqs. (87) and (160) we find $N_H \geq 65 \gg N_{\text{tot}}$. Therefore, the inhomogeneous reheating mechanism cannot be used to account for the curvature perturbation in the Universe in this model.

XI. CONCLUSIONS

We have investigated whether a massive Abelian vector field whose kinetic term is evolving during inflation can be responsible for the curvature perturbation in the Universe, without the need of a negative mass-squared.

In particular, we have studied particle production of this vector field when its kinetic term is determined by a

function, similar to the gauge kinetic function in supergravity. We assumed that the dynamics of this kinetic function is dominated by a degree of freedom which is varying during inflation; at least when the cosmological scales are exiting the horizon. In supergravity the gauge kinetic function is a holomorphic function of the fields of the theory. Since scalar fields typically obtain masses comparable to the Hubble parameter H_* due to supergravity corrections during (and after) inflation [15], one typically expects that the value of the gauge kinetic function is indeed varying during inflation, as these fields roll down the potential slopes. We parametrised these fields using a single degree of freedom ϕ , which rolls down towards its VEV: M . With respect to this degree of freedom we expressed the kinetic function as $f(\phi) = (\phi/M)^{2n}$ so that the vector field becomes canonically normalised when ϕ reaches its VEV. We have obtained the condition for the generation of a scale invariant spectrum of perturbations and showed that it can be naturally achieved when $m_\phi \sim H_*$, where m_ϕ is the tachyonic mass of ϕ . We, then studied two particular cases: i) The case of a vector field with constant mass m and ii) the case with a vector field Higgsed with ϕ , whose mass is $m = g\phi$. Then, we argued that the most promising results are obtained when $\dot{f}/f = 2H_*$ during inflation and also when $n = 1$ (i.e. $f \propto \phi^2$), which can be achieved if $m_\phi = 2H_*$. A mass of this order is naturally expected in supergravity, due to Kähler corrections to the scalar potential [15].

After obtaining a scale invariant spectrum we attempted to employ the curvaton mechanism in order to generate the observed curvature perturbation. Under this mechanism the vector field remains subdominant during inflation when it obtains a scale-invariant super-horizon spectrum of perturbations over the cosmological scales. After inflation, when the Hubble parameter decreases below its mass, the vector field begins oscillating. As shown in Ref. [10] a coherently oscillating, homogeneous, massive Abelian vector field corresponds to pressureless, *isotropic* matter, and can dominate (or nearly dominate) the Universe without introducing a long-range anisotropy. When it does so, it imprints its own curvature perturbation spectrum, as in the curvaton scenario. We followed the evolution of the vector field and obtained the corresponding bounds on the inflationary scale for the scenario to work. We found, however, that the parameter space does not allow enough inflation for the solution of the flatness and horizon problems. To overcome this problem the lower bound on the inflationary scale must be relaxed, i.e. low-scale inflation is required.

To attain low-scale inflation we employed the mass increment mechanism, first introduced in Ref. [4]. In this scenario a phase transition after the end of inflation enlarges the mass of the vector curvaton field. We have explored the parameter space under this mechanism and showed that it is possible to solve the horizon and flatness problems and also produce the required amplitude for the scale-invariant spectrum of curvature perturbations pro-

vided the phase transition does not occur much earlier than the breakdown of electroweak unification. We also found that the best results are obtained if the curvaton decays as soon as it comes to dominate the Universe.

We demonstrated our findings in a concrete example, which serves as an existence proof that the mechanism works with natural values of the parameters. In our example we considered the case of a Higgsed vector curvaton, which can also explain the initial conditions of the rolling ϕ . We have assumed that the phase transition which enlarges the mass of the field occurs at temperature ~ 0.1 TeV. The mass of the vector field is roughly comparable to the inflationary scale, which turns-out to be $V_*^{1/4} \sim 10^5$ GeV. Reheating is assumed prompt, but the reheating temperature is low enough not to result in gravitino overproduction. At the phase transition the vector curvaton assumes roughly 1 % of the density of the Universe, which is reasonable on energy equipartition grounds. Rapid oscillations of the vector field allow it to dominate the Universe at temperature ~ 1 GeV. The vector field is taken to decay at domination so as not to disturb BBN. The scenario works for $1 \text{ MeV} < M \lesssim 10^{16} \text{ GeV}$, which is a comfortably large range of parameter space, including both the grand unified and the electroweak scales. Unless one considers a large-field model of inflation the spectral index is indistinguishable from unity. However, for a large-field model (e.g. chaotic inflation), one attains a lower value for the spectral index, which agrees better with the observations.

Finally, we have also studied the possibility that the vector field generates the observed curvature perturbation spectrum through the so-called inhomogeneous reheating mechanism. In this case, the rolling ϕ field is taken to be the inflaton, whose decay rate is modulated by the perturbations in the vector field. Even though the idea sounds promising, our results show that the scenario is inviable.

In summary, we have investigated the use of a massive Abelian vector field for the generation of the observed curvature perturbation spectrum in the Universe. We have shown that, it is possible to attain a scale-invariant spectrum with a positive mass-squared for the vector field, provided the kinetic function is growing during inflation. In this case the vector field can act as a curvaton. The mechanism works with low-scale inflation, when the mass of the vector field increases at a phase transition near the breakdown of electroweak unification. The form of the kinetic function as well as other aspects of the mechanism (such as masses of order the Hubble scale) can be naturally accommodated in the theoretical framework of supergravity.

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